71. **(a)** $\frac{ds}{dt} = 64 - 32t$ $\frac{d^2s}{dt^2} = -32$
(b) 2 sec **(c)** 64 ft/sec
(d) $\frac{64}{5.2} \approx 12.3$ sec; $s\left(\frac{64}{5.2}\right) \approx 393.8$ ft

72. **(a)** $\frac{4}{7}$ sec; 280 cm/sec
(b) 560 cm/sec; 980 cm/sec^2

73. $\pi(20x - x^2)$

74. **(a)** $r(x) = \left(3 - \frac{x}{40}\right)^2 x = 9x - \frac{3}{20}x^2 + \frac{1}{1600}x^3$
(b) 40 people; \$4.00
(c) One possible answer: Probably not, since the company charges less overall for 60 passengers than it does for 40 passengers.

75. **(a)** -0.6 km/sec **(b)** $18/\pi \approx 5.73$ revolutions/min

76. **(a)** The derivative of y_1 is y_2. **(b)** Let $y_2 = \frac{|\cos(x)|}{\cos(x)}$.

77. $a = \frac{3\sqrt[3]{2}}{8}$

78. $a = \sqrt{2}$

79. **(a)** $x \neq k\frac{\pi}{4}$, where k is an odd integer
(b) $(-\pi/2, \pi/2)$
(c) Where it's not defined, at $x = k\frac{\pi}{4}$, k an odd integer
(d) It has period $\pi/2$ and continues to repeat the pattern seen in this window.

80. $y'(r) = -\frac{1}{2r^2 l}\sqrt{\frac{T}{\pi d}}$, so increasing r decreases the frequency.

$y'(l) = -\frac{1}{2rl^2}\sqrt{\frac{T}{\pi d}}$, so increasing l decreases the frequency.

$y'(d) = -\frac{1}{4rl}\sqrt{\frac{T}{\pi d^3}}$, so increasing d decreases the frequency.

$y'(T) = -\frac{1}{4rl\sqrt{\pi T d}}$, so increasing l increases the frequency.

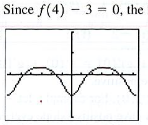
81. **(a)** $v(t) = x'(t) = 3t^2 - 12$
(b) $a(t) = v'(t) = 6t$
(c) The particle is at rest when $3t^2 - 12 = 0$; that is, at $t = 2$.
(d) $a(t) = 6t = 0$ when $t = 0$, at which point the speed is $|v(0)| = |-12| = 12$.
(e) The position at $t = 3$ is $x(3) = -4$, and the velocity is $v(3) = 15$. Since the particle is to the left of the origin and moving to the right, it is moving toward the origin.

82. **(a)** $y - 3 = 5(x - 4)$
(b) Yes. Since f is differentiable at $x = 3$, it is continuous at $x = 3$.
(c) Yes. Since f is continuous on $[2, 4]$, it takes on all values between $f(2) = -1$ and $f(4) = 3$ (Intermediate Value Theorem).
(d) $g'(2) = \frac{d}{dx}\left(\frac{f(x)}{f(x) - 3}\right)\Bigg|_{x=2}$

$= \frac{f'(2)(f(2) - 3) - (f'(2) - 0)f(2)}{(f(2) - 3)^2} = -\frac{9}{16}$.

(e) Since $f(4) - 3 = 0$, the function g is not defined at $x = 4$.

83. **(a)**



$[-2\pi, 2\pi]$ by $[-2, 2]$

(b) $f'(x) = \frac{2 \sin x}{(\cos x - 2)^2}$ **(c)** $0, \pm\pi, \pm 2\pi$
(d) The low turning points are $f(0) = f(\pm 2\pi) = \frac{1}{1 - 2} = -1$, and the high turning points are

$f(\pm\pi) = \frac{-1}{-1 - 2} = \frac{1}{3}$. The range is the interval $\left[-1, \frac{1}{3}\right]$.

CHAPTER 4

Section 4.1

Quick Review 4.1

1. $\sin(x^2 + 1)$ **3.** $49x^2 + 1$
5. $\sin \frac{x^2 + 1}{7x}$ **7.** $g(h(f(x)))$
9. $f(h(h(x)))$

Exercises 4.1

1. $3\cos(3x + 1)$
3. $-\sqrt{3}\sin(\sqrt{3}x)$
5. $\frac{2 \sin x}{(1 + \cos x)^2}$
7. $-\sin(\sin x)\cos x$
9. $3\sin\left(\frac{\pi}{2} - 3t\right)$
11. $\frac{4}{\pi}\cos 3t - \frac{4}{\pi}\sin 5t$
13. $-2(x + \sqrt{x})^{-3}\left(1 + \frac{1}{2\sqrt{x}}\right)$
15. $-5\sin^{-6}x\cos x + 3\cos^2 x \sin x$
17. $4\sin^3 x \sec^2 4x + 3\sin^2 x \cos x \tan 4x$
19. $-3(2x + 1)^{-3/2}$
21. $6\sin(3x - 2)\cos(3x - 2) = 3\sin(6x - 4)$
23. $-42(1 + \cos^2 7x)^2 \cos 7x \sin 7x$
25. $-\sec^2(2 - \theta)$
27. $\frac{\theta\cos\theta + \sin\theta}{2\sqrt{\theta\sin\theta}}$
29. $2\sec^2 x \tan x$
31. $18\csc^2(3x - 1)\cot(3x - 1)$
33. $5/2$
35. $-\pi/4$
37. 0
39. **(a)** $-6\sin(6x + 2)$
(b) $-6\sin(6x + 2)$
41. $y = -x + 2\sqrt{2}$
43. $y = -\frac{1}{2}x - \frac{1}{2}$
45. $y = x + \frac{1}{4}$
47. $y = \sqrt{3}x + 2 - \frac{\pi}{\sqrt{3}}$

49. **(a)** $\frac{\cos t}{2t + 1}$
(b) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = -\frac{(2t + 1)(\sin t) + 2\cos t}{(2t + 1)^2}$
(c) $\frac{d}{dx}\left(\frac{dy}{dx}\right) = -\frac{(2t + 1)(\sin t) + 2\cos t}{(2t + 1)^3}$
(d) part (c)

51. 5

53. $\frac{1}{2}$

55. Tangent $y = \pi x - \pi + 2$: Normal: $y = -\frac{1}{\pi}x + \frac{1}{\pi} + 2$

57. $\frac{d}{dx}\cos(x°) = \frac{d}{dx}\cos\left(\frac{\pi x}{180}\right) = -\frac{\pi}{180}\sin\left(\frac{\pi x}{180}\right) = -\frac{\pi}{180}\sin(x°)$

59. The slope of $y = \sin(2x)$ at the origin is 2. The slope of $y = -\sin\frac{x}{2}$ at the origin is $-\frac{1}{2}$. So the lines tangent to the two curves at the origin are perpendicular.

61. The amplitude of the velocity is doubled.
The amplitude of the acceleration is quadrupled.
The amplitude of the jerk is multiplied by 8.

63. Velocity $= \frac{2}{5}$ m/sec

acceleration $= -\frac{4}{125}$ m/sec^2

65. Given: $v = \frac{k}{\sqrt{s}}$

acceleration: $= \frac{dv}{dt} = \frac{dv}{ds}\frac{dv}{dt} = \frac{dv}{ds}v$

$= \frac{-k}{2s^{3/2}}\frac{k}{\sqrt{s}} = -\frac{k^2}{2s^2}$

67. $\frac{dT}{du} = \frac{dT}{dL}\frac{dL}{du}$

$= \frac{\pi}{\sqrt{gL}}kL = k\pi\sqrt{\frac{L}{g}} = \frac{kT}{2}$

69. Yes. Either the graph of $y = g(x)$ must have a horizontal tangent at $x = 1$, or the graph of $y = f(u)$ must have a horizontal tangent at $u = g(1)$. This is because $\frac{d}{dx}f(g(x)) = f'(g(x))\,g'(x)$, so the slope of the tangent to the graph of $y = f(g(x))$ at $x = 1$ is given by $f'(g(1))\,g'(1)$. If this product is zero, then at least one of its factors must be zero.

71. False. It is $+1$

73. C

75. B

77. As $h \to 0$, the second curve (the difference quotient) approaches the first $(y = -2x\sin(x^2))$. This is because $-2x\sin(x^2)$ is the derivative of $\cos(x^2)$, and the second curve is the difference quotient used to define the derivative of $\cos(x^2)$. As $h \to 0$, the difference quotient expression should be approaching the derivative.

79. $\frac{dG}{dx} = \frac{d}{dx}\sqrt{uv} = \frac{d}{dx}\sqrt{x^2 + cx} = \frac{2x + c}{2\sqrt{x^2 + cx}}$

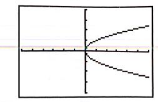
$= \frac{x + \frac{c}{2}}{\sqrt{x^2 + cx}}$

$= \frac{A}{G}$, since $A = x + \frac{c}{2}$.

Section 4.2

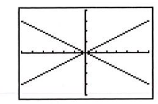
Quick Review 4.2

1. $y_1 = \sqrt{x}$, $y_2 = -\sqrt{x}$



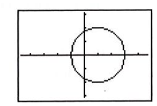
$[-6, 6]$ by $[-4, 4]$

3. $y_1 = \frac{x}{2}$, $y_2 = -\frac{x}{2}$



$[-6, 6]$ by $[-4, 4]$

5. $y_1 = \sqrt{2x + 3 - x^2}$, $y_2 = -\sqrt{2x + 3 - x^2}$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

7. $y' = \frac{y + y\cos x}{\sin x - y}$

9. $x^{3/2} - x^{5/6}$

Exercises 4.2

1. $-\frac{2xy + y^2}{2xy + x^2}$

3. $\frac{1}{y(x + 1)^2}$

5. $\cos^2 y$

7. $-\frac{1}{x}\cos^2(xy) - \frac{y}{x}$

9. $\frac{dy}{dx} = -\frac{x}{y}$, $2/3$

11. $\frac{dy}{dx} = -\frac{x - 1}{y - 1}$, $-2/3$

13. $\frac{dy}{dx} = \frac{2xy - y^2}{2xy - x^2}$, defined at every point except where $x = 0$ or $y = x/2$

15. $\frac{dy}{dx} = \frac{3x^2 - y}{x - 3y^2}$, defined at every point except where $y^2 = x/3$

17. **(a)** $y = \frac{7}{4}x - \frac{1}{2}$ **(b)** $y = -\frac{4}{7}x + \frac{29}{7}$

19. **(a)** $y = 3x + 6$ **(b)** $y = -\frac{1}{3}x + \frac{8}{3}$

21. **(a)** $y = \frac{6}{7}x + \frac{6}{7}$
(b) $y = -\frac{7}{6}x - \frac{7}{6}$

23. (a) $y = -\frac{\pi}{2}x + \pi$ (b) $y = \frac{2}{\pi}x - \frac{2}{\pi} + \frac{\pi}{2}$

25. (a) $y = 2\pi x - 2\pi$ (b) $y = -\frac{x}{2\pi} + \frac{1}{2\pi}$

27. $\frac{dy}{dx} = -\frac{x}{y}$

$\frac{d^2y}{dx^2} = -\frac{(x^2 + y^2)}{y^3} = -\frac{1}{y^3}$

29. $\frac{dy}{dx} = \frac{x + 1}{y}$

$\frac{d^2y}{dx^2} = -\frac{y^2 - (x + 1)^2}{y^3} = -\frac{1}{y^3}$

31. $(9/4)x^{5/4}$

33. $(1/3)x^{-2/3}$

35. $-(2x + 5)^{-3/2}$

37. $x^2(x^2 + 1)^{-1/2} + (x^2 + 1)^{1/2}$

39. $-\frac{1}{4}(1 - x^{1/2})^{-1/2}x^{-1/2}$

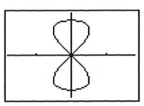
41. $-\frac{9}{2}(\csc x)^{3/2} \cot x$

43. (b), (c), and (d)

45. (a) At $\left(\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{2}\right)$: Slope $= -1$;

at $\left(\frac{\sqrt{3}}{4}, \frac{1}{2}\right)$: Slope $= \sqrt{3}$

(b)



[-1.8, 1.8] by [-1.2, 1.2]

Parameter interval:
$-1 \le t \le 1$

47. (a) $(-1)^3(1)^2 = \cos(\pi)$ is true since both sides equal: -1.
(b) The slope is 3/2.

49. The points are $(\pm\sqrt{7}, 0)$

$\frac{dy}{dx} = -\frac{2x + y}{2y + x}$

At both points, $\frac{dy}{dx} = -2$

51. First curve: $\frac{dy}{dx} = -\frac{2x}{3y}$

second curve: $\frac{dy}{dx} = \frac{3x^2}{2y}$

At (1, 1), the slopes are $-\frac{2}{3}$ and $\frac{3}{2}$ respectively.

At (1, −1), the slopes are $\frac{2}{3}$ and $-\frac{3}{2}$ respectively.

In both cases, the tangents are perpendicular.

53. Acceleration $= \frac{dv}{dt} = 4(s - t)^{-1/2}(v - 1)$

$= 32$ ft/sec^2

55. (a) At (4, 2): $\frac{5}{4}$; at (2, 4): $\frac{4}{5}$

(b) At $(3\sqrt[3]{2}, 3\sqrt[3]{4}) \approx (3.780, 4.762)$

(c) At $(3\sqrt[3]{4}, 3\sqrt[3]{2}) \approx (4.762, 3.780)$

57. At (−1, −1): $y = -2x - 3$; at (3, −3: $y = -2x + 3$

59. False. It is equal to −2.

61. A **63.** E

65. (a) $\frac{dy}{dx} = -\frac{b^2x}{a^2y}$

The tangent line is $y - y_1 = -\frac{b^2x_1}{a^2y_1}(x - x_1)$.

This gives: $a^2y_1y - a^2y_1^2 = -b^2x_1x + b^2x_1^2$,

$a^2y_1y + b^2x_1x = a^2y_1^2 + b^2x_1^2$.

But $a^2y_1^2 + b^2x_1^2 = a^2b^2$ since (x_1, y_1) is on the ellipse.

Therefore, $a^2y_1y + b^2x_1x = a^2b^2$, and dividing by a^2b^2 gives

$\frac{x_1x}{a^2} + \frac{y_1y}{b^2} = 1$. (b) $\frac{x_1x}{a^2} - \frac{y_1y}{b^2} = 1$.

Quick Quiz (Sections 4.1–4.2)

1. B **3.** C

Section 4.3

Quick Review 4.3

1. Domain: $[-1, 1]$; Range: $[-\pi/2, \pi/2]$ At 1: $\pi/2$

3. Domain: all reals Range: $(-\pi/2, \pi/2)$ At 1: $\pi/4$

5. Domain: all reals; Range: all reals At 1: 1

7. $f^{-1}(x) = x^3 - 5$

9. $f^{-1}(x) = \frac{2}{3 - x}$

Exercises 4.3

1. $-\frac{2x}{\sqrt{1 - x^4}}$ **3.** $\frac{\sqrt{2}}{\sqrt{1 - 2t^2}}$

5. $-\frac{6}{t\sqrt{t^4 - 9}}$ **7.** $\sin^{-1} x$

9. $\sqrt{7}/7$ **11.** 1/5

13. $\frac{1}{|2s + 1|\sqrt{s^2 + s}}$

15. $-\frac{2}{(x^2 + 1)\sqrt{x^2 + 2}}$

17. $-\frac{1}{\sqrt{1 - t^2}}$ **19.** $-\frac{1}{2t\sqrt{t - 1}}$

21. $0, x > 1$ **23.** $y = 0.289x + 0.470$

25. $y = 0.378x - 0.286$

27. (a) $y = 2x - \frac{\pi}{2} + 1$

(b) $y = \frac{1}{2}x - \frac{1}{2} + \frac{\pi}{4}$

29. (a) $f'(x) = 3 - \sin x$ and $f'(x) \ne 0$. So f has a differentiable inverse by Theorem 3.

(b) $f(0) = 1, f'(0) = 3$

(c) $f^{-1}(1) = 0, (f^{-1})'(1) = \frac{1}{3}$

31. (a) $v(t) = \frac{dx}{dt} = \frac{1}{1 + t^2}$ which is always positive.

(b) $a(t) = \frac{dv}{dt} = -\frac{2t}{(1 + t^2)^2}$ which is always negative.

(c) $\frac{\pi}{2}$

33. $\frac{d}{dx}\cot^{-1} x = \frac{d}{dx}\left(\frac{\pi}{2} - \tan^{-1} x\right)$

$= 0 - \frac{d}{dx}\tan^{-1} x$

$= -\frac{1}{1 + x^2}$

35. True. By definition of the function.

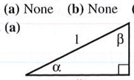
37. E **39.** A

41. (a) $y = \pi/2$ (b) $y = -\pi/2$ (c) None

43. (a) $y = \pi/2$ (b) $y = \pi/2$ (c) None

45. (a) None (b) None (c) None

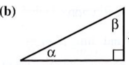
47. (a)



$\alpha = \cos^{-1} x, \beta = \sin^{-1} x$

So $\pi/2 = \alpha + \beta = \cos^{-1} x + \sin^{-1} x$.

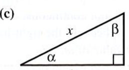
(b)



$\alpha = \tan^{-1} x, \beta = \cot^{-1} x$

So $\pi/2 = \alpha + \beta = \tan^{-1} x + \cot^{-1} x$.

(c)



$\alpha = \sec^{-1} x, \beta = \csc^{-1} x$

So $\pi/2 = \alpha + \beta = \sec^{-1} x + \csc^{-1} x$.

49. If s is the length of a side of the square, and let α, β, γ denote the angles labeled $\tan^{-1} 1$, $\tan^{-1} 2$, and $\tan^{-1} 3$, respectively.

$\tan\alpha = \frac{s}{s} = 1$, so $\alpha = \tan^{-1} 1$ and

$\tan\beta = \frac{s}{s/2} = 2$, so $\beta = \tan^{-1} 2$.

$\gamma = \pi - \alpha - \beta = \pi - \tan^{-1} 1 - \tan^{-1} 2$

$= \tan^{-1} 3$.

Section 4.4

Quick Review 4.4

1. $\frac{\ln 8}{\ln 5}$ **3.** $\tan x$

5. $3x - 15$ **7.** $\ln(4x^4)$

9. $t = \frac{\ln 18 - \ln(\ln 5)}{\ln 5} \approx 1.50$

Exercises 4.4

1. $2e^x$ **3.** $-e^{-x}$

5. $\frac{2}{3}e^{2x/3}$ **7.** $e^2 - e^x$

9. $e^{\sqrt{x}}/(2\sqrt{x})$ **11.** $8^x \ln 8$

13. $-3^{\csc x}(\ln 3)(\csc x \cot x)$

15. $\frac{2}{x}$ **17.** $-\frac{1}{x}, x > 0$

19. $\frac{1}{x \ln x}$ **21.** $\frac{2}{x \ln 4} = \frac{1}{x \ln 2}$

23. $-\frac{1}{x \ln 2}, x > 0$ **25.** $\frac{1}{x}, x > 0$

27. $\frac{1}{\ln 10}$ **29.** $\approx (1.379, 5.551)$

31. $2e^{-1}$

33. $\pi x^{\pi - 1}$

35. $-\sqrt{2}x^{-\sqrt{2}-1}$

37. $\frac{1}{x + 2}, x > -2$

39. $\frac{\sin x}{2 - \cos x}$, all reals

41. $\frac{3}{(3x + 1)\ln 2}, x > -1/3$

43. $(\sin x)^x[x \cot x + \ln(\sin x)]$

45. $\left(\frac{(x - 3)^4(x^2 + 1)}{(2x + 5)^3}\right)^{1/5}\left(\frac{4}{5(x - 3)} + \frac{2x}{5(x^2 + 1)} - \frac{6}{5(2x + 5)}\right)$

47. $\frac{2x^{\ln x}(\ln x)}{x}$

49. $y = ex$

51. (a) 18
(b) 52 students per day
(c) After 4 days; 52 students per day

53. rate $\approx$ 0.098 grams/day

55. (a) ln 2

(b) $f'(0) = \lim_{h \to 0} \frac{2^h - 1}{h}$

(c) ln 2
(d) ln 7

57. False. It is $(\ln 2)2^x$.

59. B

61. A

63. (a) The graph of y_4 is a horizontal line at $y = a$.
(b) The graph of y_3 is a horizontal line at $y = \ln a$.
(c) $\frac{d}{dx}a^x = a^x$ if and only if $y_3 = \frac{y_2}{y_1} = 1$.

So if $y_3 = \ln a$, then $\frac{d}{dx}a^x$ will equal a^x if and only if $\ln a = 1$, or $a = e$.

(d) $y_2 = \frac{d}{dx}a^x = a^x \ln a$. This will equal $y_1 = a^x$ if and only if $\ln a = 1$, or $a = e$.

65. (a) $y = \frac{1}{e}x$
(b) Because the graph of $\ln x$ lies below the graph of the tangent line for all positive $x \ne e$.
(c) Multiplying by e, $e(\ln x) < x$, or $\ln x^e < x$.
(d) Exponentiate both sides of the inequality in part (c).
(e) Let $x = \pi$ to see that $\pi^e < e^\pi$.

Quick Quiz (Sections 4.3–4.4)

1. A
3. C

Review Exercises

1. $3e^{3x-7}$ **2.** $e^x\sec^2(e^x)$ **3.** $3\sin^2 x\cos x$

4. $-\cot x$ **5.** $2\sin(1-2t)$ **6.** $\frac{2}{t^2}\csc^2\frac{2}{t}$

7. $-\frac{\sin x}{2\sqrt{1+\cos x}}$

8. $\frac{3x+1}{\sqrt{2x+1}}$

9. $3\sec(1+3\theta)\tan(1+3\theta)$

10. $-4\theta\tan(3-\theta^2)\sec^2(3-\theta^2)$

11. $-5x^2\csc 5x\cot 5x + 2x\csc 5x$

12. $\frac{1}{2x}, x>0$ **13.** $\frac{e^x}{1+e^x}$

14. $-xe^{-x}+e^{-x}$ **15.** e

16. $\cot x$, where x is an interval of the form $(k\pi, (k+1)\pi)$, k even

17. $-\frac{1}{\cos^{-1}x\sqrt{1-x^2}}$

18. $\frac{2}{\theta\ln 2}$

19. $\frac{1}{(t-7)\ln 5}, t>7$

20. $-8^{-t}\ln 8$

21. $\frac{2(\ln x)x^{\ln x}}{x}$

22. $\frac{(2\cdot 2^x)[x^3\ln 2 + x\ln 2 + 1]}{(x^2+1)^{3/2}}$ or

$\frac{(2x)2^x}{\sqrt{x^2+1}}\left(\frac{1}{2}+\ln 2-\frac{x}{x^2+1}\right)$

23. $\frac{e^{\tan^{-1}x}}{1+x^2}$

24. $-\frac{u}{\sqrt{u^2-u^4}} = -\frac{u}{|u|\sqrt{1-u^2}}$

25. $\frac{t}{|t|\sqrt{t^2-1}} + \sec^{-1}t - \frac{1}{2t}$

26. $-\frac{2+2t^2}{1+4t^2} + 2t\cot^{-1}2t$

27. $\cos^{-1}z$

28. $-\frac{1}{x} + \frac{\csc^{-1}\sqrt{x}}{\sqrt{x-1}}$

29. -1

30. $2\left(\frac{1+\sin\theta}{1-\cos\theta}\right)\left(\frac{\cos\theta-\sin\theta-1}{(1-\cos\theta)^2}\right)$

31. For all $x \neq 0$

32. For all real x

33. For all $x<1$

34. For all $x \neq 0$

35. $-\frac{y+2}{x+3}$

36. $-\frac{1}{3}(xy)^{-1/5}$

37. $-\frac{y}{x}$ or $-\frac{1}{x^2}$

38. $\frac{1}{2y(x+1)^2}$

39. $-\frac{2x}{y^5}$ **40.** $\frac{1+2xy^2}{x^4y^3}$

41. $-2\frac{(3y^2+1)^2\cos x + 12y\sin^2 x}{(3y^2+1)^3}$

42. $\frac{2}{3}x^{-4/3}y^{1/3} + \frac{2}{3}x^{-5/3}y^{2/3} = \frac{8}{3}x^{-5/3}y^{1/3}$

43. $32e^{\sqrt[8]{2x}}$

44. $y = 32\sin(\sqrt[8]{2x})$

45. (a) $y = \frac{2}{\sqrt{3}}x - \sqrt{3}$ (b) $y = -\frac{\sqrt{3}}{2}x + \frac{5\sqrt{3}}{2}$

46. (a) $y+\sqrt{3} = 8\left(x-\frac{\pi}{3}\right)$ (b) $y+\sqrt{3} = -\frac{1}{8}\left(x-\frac{\pi}{3}\right)$

47. (a) $y = -\frac{1}{4}x+\frac{9}{4}$ (b) $y = 4x-2$

48. (a) $y = -\frac{5}{4}x+6$ (b) $y = \frac{4}{5}x - \frac{11}{5}$

49. $y = x - 2\sqrt{2}$

50. $y = \frac{4}{3}x + 4\sqrt{2}$

51. $y = \frac{10}{3}x - 5\sqrt{3}$

52. $y = (1+\sqrt{2})x - \sqrt{2} - 1 - \frac{\pi}{4}$ or $y \approx 2.414x - 3.200$

53. (a) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(\sin ax + b\cos x) = b$ and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(5x+3) = 3$. Thus $\lim_{x\to 0} f(x) = f(0) = 3$ if and only if $b = 3$

(b) $f'(x) = \begin{cases} a\cos ax - b\sin x, & x<0 \\ 5, & x>0 \end{cases}$. The slopes match at $x=0$ if and only if $a = 5$.

(c) No. Although the slopes match, the function is not continuous.

54. (a) The function is continuous for all values of m, because the right-hand limit as $x\to 0$ is equal to $f(0) = 0$ for any value of m.

(b) The left-hand derivative at $x=0$ is 2, and the right-hand derivative at $x=0$ is m, so in order for the function to be differentiable at $x=0$, m must be 2.

55. (a) For all $x \neq 1$ (b) At $x=1$ (c) Nowhere

56. (a) For all x (b) Nowhere (c) Nowhere

57. (a) $[-1,1)\cup(1,3]$ (b) At $x=1$ (c) Nowhere

58. (a) $[-3,3]$ (b) Nowhere (c) Nowhere

59. (a) $-\cos x$ (b) $14x-13$ (c) $2t-3$ (d) $2t-30t^5$

60. (a) $\frac{2}{2x+7} - \frac{3}{3x+2}$ (b) 1

(c) $-2t$ (d) $\frac{80}{3\sqrt[3]{t}}$

61. (a) $y-4 = -\frac{1}{3}(x-2)$

(b) $y-2 = \frac{1}{3}(x-4)$

(c) $y-2 = \frac{1}{2}(x-2)$

62. (a) $y-3 = \frac{1}{2}(x-1)$

(b) $y-1 = -\frac{1}{2}(x-3)$

(c) $y-3 = -4(x-1)$

63. $\frac{dy}{dx} = \frac{(x+2)^5(2x-3)^4}{(x+17)^2}\left(\frac{5}{x+2}+\frac{8}{2x+3}-\frac{2}{x+17}\right)$

64. $\frac{dy}{dx} = (x^2+2)^{x+5}\left(\ln(x^2+2)+\frac{2x^2+10x}{x^2+2}\right)$

65. (a) $f(x) = \frac{x^2}{2}$ or $f(x) = \frac{x^2}{2}+C$

(b) $f(x) = e^x$ or $f(x) = Ce^x$

(c) $f(x) = e^{-x}$ or $f(x) = Ce^{-x}$

(d) $f(x) = e^x$ or $f(x) = e^{-x}$ or $f(x) = Ce^x + De^{-x}$

(e) $f(x) = \sin x$ or $f(x) = \cos x$ or $f(x) = C\sin x + D\cos x$

66. (a) $-13/10$ (b) $-1/3$
(c) $1/10$ (d) -1
(e) $-2/3$ (f) -12

67. (a) 5 (b) 0
(c) 8 (d) 2
(e) 4 (f) -1

68. $\sqrt{3}$ **69.** $-1/6$

70. (a) One possible answer:

$x(t) = 10\cos\left(t+\frac{\pi}{4}\right)$

$y(t) = 0$

(b) $5\sqrt{2}$

(c) $s = -10$ and $s = 10$

(d) At $t = \pi/4$:
Velocity $= -10$
Speed $= 10$
Acceleration $= 0$

71. (a) $A(-1,1)$; $B(1,-1)$
(b) $C(-0.5,2)$; $D(0.5,-2)$

72. (a) $A(-\sqrt{2},-2\sqrt{2})$; $B(\sqrt{2},2\sqrt{2})$
(b) $C(-2,-2)$; $D(2,2)$

73. (a) $A(-2,-2)$
(b) $B(-1,-3)$

74. At y-intercept $(0,2\sqrt{2})$ the slope is $\frac{2+\sqrt{2}}{2}$.

At y-intercept $(0,-2\sqrt{2})$ the slope is $\frac{2-\sqrt{2}}{2}$.

At x-intercept $(2+2\sqrt{3},0)$ the slope is $\frac{\sqrt{3}}{\sqrt{3}+1}$.

At x-intercept $(2-2\sqrt{3},0)$ the slope is $\frac{\sqrt{3}}{\sqrt{3}-1}$.

75. 6

76. Every sinusoid with amplitude A and period p is the graph of some equation of the form $y = A\sin\left(\frac{2\pi}{p}x+k\right)+D$. The slope at any x is $\frac{dy}{dx} = A\cdot\frac{2\pi}{p}\cos\left(\frac{2\pi}{p}x+k\right)$. Since the maximum value of cosine is 1, the maximum slope is $\frac{2\pi A}{p}$.

77. Yes

78. (a) $P(0) \approx 1.339$, so initially, one student was infected
(b) 200
(c) After 5 days, when the rate is 50 students/day

79. (a) $-\frac{2}{3}$

(b) $-\frac{5}{27}$

80. $-1/(3\sqrt{3})$

81. (a) $g'(x) = k\cdot e^{kx} + f'(x)$, so $g'(0) = k+3$.
$g''(x) = k^2\cdot e^{kx} + f''(x)$, so $g''(0) = k^2 - 1$.
(b) $h'(x) = b\sin(bx)f(x) + f'(x)\cos(bx)$, so $h'(0) = b\cdot\sin(0) + 3\cdot\cos(0) = 3$. Note that $h(0) = \cos(0)\cdot f(0) = 1\cdot 2 = 2$, so the tangent line has equation $y - 2 = 3(x-0)$.

82. (a) $\frac{dy}{dx} = \frac{e^x - e^{-x}}{2}$

(b) $\frac{d^2y}{dx^2} = \frac{e^x+e^{-x}}{2}$

(c) At $x = 1$, $\frac{dy}{dx} = \frac{e^1-e^{-1}}{2} = 1.175$ and $y = \frac{e^1+e^{-1}}{2} = 1.543$. The tangent line has equation $y - 1.543 = 1.175(x-1)$.

(d) The normal line has equation $y - 1.543 = -0.851(x-1)$.

(e) The tangent line is horizontal where $dy/dx = 0$; that is, where $e^x = e^{-x}$. This is true only at $x = 0$.

83. (a) The domain of f is the interval $(-1,1)$.

(b) $f'(x) = \frac{-2x}{1-x^2} = \frac{2x}{x^2-1}$ on the domain $(-1,1)$.

(c) The domain of f' is the interval $(-1,1)$.

(d) $f''(x) = \left(\frac{2x}{x^2-1}\right)' = \frac{2(x^2-1)-2x(2x)}{(x^2-1)^2} = \frac{-2(x^2+1)}{(x^2-1)^2} < 0$ for all x in the domain of f, since $\frac{x^2+1}{(x^2-1)^2} > 0$ for all x between -1 and 1.

CHAPTER 5

Section 5.1

Quick Review 5.1

1. $\frac{-1}{2\sqrt{4-x}}$ **3.** $\frac{\sin(\ln x)}{x}$

5. (c) **7.** (d) **9.** ∞

11. (a) 1 (b) 1 (c) Undefined

Exercises 5.1

1. Minima at $(-2,0)$ and $(2,0)$, maximum at $(0,2)$

3. Maximum at $(0,5)$

5. Maximum at $x = b$, minimum at $x = c_2$; Extreme Value Theorem applies, so both the max and min exist.

7. Maximum at $x = c$, no minimum; Extreme Value Theorem doesn't apply, since the function isn't defined on a closed interval.

9. Maximum at $x = c$, minimum at $x = a$; Extreme Value Theorem doesn't apply, since the function isn't continuous.

11. Maximum value is $\frac{1}{4} + \ln 4$ at $x = 4$; minimum value is 1 at $x = 1$; local maximum at $\left(\frac{1}{2}, 2-\ln 2\right)$

13. Maximum value is $\ln 4$ at $x = 3$; minimum value is 0 at $x = 0$.

15. Maximum value is 1 at $x = \frac{\pi}{4}$; minimum value is -1 at $x = \frac{5\pi}{4}$; local minimum at $\left(0, \frac{1}{\sqrt{2}}\right)$; local maximum at $\left(\frac{7\pi}{4}, 0\right)$

17. Maximum value is $3^{2/5}$ at $x = -3$; minimum value is 0 at $x = 0$. [(0, 0) is not a stationary point]

19. Min value 1 at $x = 2$